THE IMPORTANCE OF MOMENTUM TRANSFER IN COLLISION-INDUCED BREAKUPS IN LOW EARTH ORBIT

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INTRODUCTION

Although there is adequate information on larger objects in low Earth orbit, specifically those objects larger than about 10 cm in diameter, there is little direct information on objects from this size down to 1 millimeter. Yet this is the size regime where objects acting as projectiles represent the ability to seriously damage or destroy a functioning spacecraft if they collide with it. Since there is poor data in this size regime, this population component must be inferred from the creation of larger fragments in observed breakups. Of the three commonly attributed causes of breakup, low- or high-intensity explosions and collisions /1/, only collisions, with a power law distribution in fragment size, represents the potential for a significant source of millimeter and centimeter debris.

The observed consequences of known collisional breakups in orbit indicates no significant momentum transfer in the resulting debris cloud. The position taken in this paper is that this is an observational selection effect, that what is seen in these events is an explosion-like breakup of the target structure arising from shock waves introduced into the structure by the collision, but one that occurs significantly after the collision processes are completed; the collision cloud, in which there is momentum transfer, consists of small, unobserved fragments. Preliminary computations of the contribution of one known collisional breakup, Solwind at 500 km in 1985, and Cosmos 1275 at 950 km in 1981, assume no momentum transfer on breakup and indicate that these 2 events are the dominant contributors to the current millimeter and centimeter population. A different story would emerge if momentum transfer was taken into account.

The establishment of the role of momentum transfer in collisional processes will become more critical in the future, as collisions become more frequent. Also, kinetic energy anti-satellite (ASAT) weapons tests and usage, which might be anticipated, need to be understood in the role they will play in the state of the environment.

DISCUSSION

Observation of On-Orbit Collisional Breakups

There are 2 cases in which collisional breakups have occurred in orbit under known conditions. The first of these was the test of a hovering ASAT vehicle by the United States in 1985 using as a target Solwind, a science satellite. This breakup occurred at an altitude of 525 km, and the resulting debris cloud was well observed. The second was the designed impact of the upper stage and the science payload in the Delta 180 flight. Again, the debris clouds resulting from this test were well observed.

The most significant feature in both tests was that the debris clouds, a single cloud for Solwind, and two clouds for Delta 180, showed little evidence of momentum transfer occurring during the collision process/2,3/. The center of mass for the Solwind cloud was that of the satellite had it not encountered the ASAT vehicle; for the Delta 180 experiment, there were 2 debris clouds, one moving in the orbit of each vehicle. Instruments that were able to observe smaller fragments found more indication of momentum transfer than those seeing only the largest objects.

These observed results appear to oppose what seems to be intuitively obvious, that there must be momentum transfer in a collisional process. This can be viewed as demonstrating the special characteristics of hypervelocity impact processes. In laboratory tests of small projectiles at small targets, an exiting debris plume is observed if the projectile is large enough to penetrate through the target. This debris plume shows a mixture of target and projectile material where there has been momentum coupling, but consists entirely of very small particles. Scaling the interacting particles to sizes of objects in orbit, but moving the impact 100's of kilometers away would lead to a debris cloud that would be difficult to detect.

These data can be combined into a single model for collisional breakup of objects in orbit if the collisional process is viewed as directly involving only the material in the line of flight of the impacting projectile. It is this material which is subject to momentum exchange and fragment creation following the power law size distribution characterizing collisional impacts. The impact, because it is occurring at speeds higher than the sound speed in the target structure, deposits significant energy, but little momentum, in the form of shock waves propagating through the structure. The energy of these shock waves yields the catastrophic fragmentation of the entire structure that was observed in Solwind.

Because the source of breakup energy is being supplied by shock waves, it might be expected that the size distribution of this second cloud would resemble that of a high-intensity explosion, and because little momentum was transferred into these shock waves, the resulting breakup would have the motion of the unperturbed structure motion as its center of mass motion.

Model for Momentum Transfer

The model for momentum transfer is summarized in Figures 1 and 2. First of all, a spacecraft can be viewed as consisting of several weakly connected components, called elements, as shown in Figure 1. In this figure, a generic spacecraft consists of 4 elements: a main body, 2 solar panels, and an antenna. If the line of flight of debris hitting the spacecraft does not go through 2 or more elements, which it generally will not, only a single element would need to be considered in the collision process. In contrast to connections between elements, which are relatively weak, the connections within an element are strong and an element can be viewed as a single cohesive object. Within the element, the material in the target structure is viewed as having 2 components, the material in the line of flight of the projectile, denoted as the column mass, and material out of the line of flight, called the residual mass. This type of model has been suggested by Chobotov and co-workers /4/. The column mass participates in the creation of collisional debris - it plus the projectile mass yield a debris cloud having size and velocity distribution characteristics of collisional processes. If the residual mass is large enough, that material remains intact; if not, it breaks up in a size and velocity distribution characteristic of an explosion event /5/.

When hypervelocity impact occurs, a cone of ejecta, consisting of both projectile and target material, emerges from the impact site. If there is a void on the back side of the first surface in the target, as there is for a Hubble shield and for fuel tanks on spent stages, the debris cloud expands as it propagates and spreads over a larger area on the second surface it encounters. However, if there is material behind the front surface, it might be expected that this material will collimate the debris cloud, since there is no significant source of energy to cause it to expand. In effect, for filled volumes, the geometry for propagation of hypervelocity fragments is similar to that for subsonic propagation. The coupling between the collisionally involved material and the rest of the structure comes from the edge effects, where very large impulsive loads transform into shearing, with little momentum transfer. Because this coupling involves area to mass effects,

laboratory tests on scale models will have to be interpreted with care.

Within the columnar mass and the projectile, the interaction is taken to be completely inelastic, so that the consequent mass forms a single debris cloud moving with a center of mass characterized by the center of mass of the projectile plus the column mass.

The residual mass plus any appendages will remain after the collision, but will experience shock waves propagated by the initial impact. These shock waves will transform into stress waves at free surfaces, and at all changes in material conditions. Links between other elements and that directly involved in the collision will be relatively weak, and will be the most easily broken. In fact, compared to the connection strengths within these other elements, it might be expected that very little shock is propagated into these elements and that they retain their integrity, either associated with the affected structure or appearing as large debris objects emerging from the collision.

Within the element directly involved with the collision there will be much stronger bonding between the components, so there will be much more damage caused by the shock waves. Rather than viewing the consequences of impact in terms of shock waves in this structure, it is easier to picture the fragmentation of this structure as occurring from a large amount of energy being released within the structure, as would occur in an explosion.

This leads to the picture in Figure 2 of two types of clouds - one characterized by collision processes and the other by explosion processes. This is an adequate picture for the impact of objects of significantly different size. If the objects are of comparable size, since off-center collisions are most likely to occur, the overlap masses will become the column masses, and the non-overlap masses will become the residual mass. In this case there would be two explosion clouds created, one from each of the residual masses, as well as a collision cloud. This case has been discussed by Chobotov and co-workers /4/.

Velocity Space Representation of Breakup Clouds

The velocity space representation provides a singularly simple means of representing the intact objects before collision, and the debris cloud(s) after collision. If the coordinate axes in this space are taken to be Z-axis radial velocity, X-axis the in-plane horizontal velocity, and Y-axis cross-range horizontal velocity, kinematically interesting

characteristics can be expressed in terms of conic surfaces. At a given altitude, the surfaces of constant perigee altitude will be hyperboloids of revolution about the Z-axis and will have the functional form

$$v_H^2\left(\frac{r_p + r_0}{2\mu}\left(\frac{r_0}{r_p}\right)\right) - v_z^2\left(\frac{r_0\,r_p}{2\mu(r_0 - r_p)}\right) = 1 \qquad (1)$$

where

v_H = horizontal velocity = $sqrt(v_x^2 + v_y^2)$
v_z = radial velocity
r_P = perigee radius = perigee altitude + radius of Earth
r_0 = radius distance of reference point
μ = gravitational constant * mass of Earth

The associated surfaces of constant apogee altitude in this space are ellipses, which have the functional form

$$v_H^2\left(\frac{r_0 + r_0}{2\mu}\left(\frac{r_0}{r_a}\right)\right) + v_z^2\left(\frac{r_0\,r_0}{2\mu(r_a - r_0)}\right) = 1 \qquad (2)$$

where

r_A = apogee radius

Figure 3 presents a 2-dimensional cross-section of this space, with the radial velocity plotted on the vertical axis, and the horizontal velocity plotted on the horizontal axis. The space is symmetric about both the horizontal and vertical axes, so only one quadrant is shown. The altitude is 500 km. A hyperbolae opening to the right represent families of orbits having common perigee altitude, as labeled on each curve. The ellipses opening to the left are the lines of constant apogee, also labeled on each curve. The zero energy surface, marking the limit of bound orbits is the circle of radius 10.8 km/sec. The point representing a circular orbit at 500 km is indicated by the "x" at a horizontal velocity of 7.626 km/sec.

As the perigee altitude approaches 500 km (from below), the hyperbola representing orbits of that altitude become more elongated along the horizontal axis; the horizontal axis to the right of the circular orbit velocity represents orbits having perigee of 500 km. Similarly, the horizontal axis to the left of the circular orbit velocity represent orbits having apogee of 500 km.

It is obvious from this figure that there is a compression in the hyperbolae along the horizontal axis, indicating that a small reduction in the horizontal velocity for an object in circular orbit will significantly reduce the perigee altitude of the resulting orbit. A debris cloud, which forms a volume in this space, will consist of objects with orbits of lower perigee altitude, and hence reduced lifetime, if the center of mass for the cloud can be moved to the left.

The explosive breakup of a single object in orbit will retain the orbit of that object for its center of mass. That is, for an explosive breakup spherically symmetric in the co-moving frame of the exploding object, the breakup cloud will form a spherical volume in velocity space centered on the velocity of that object.

However, the picture will be different for collisionally induced breakups. Momentum exchange in a collisionally induced debris cloud will have the center of mass of the interacting material as its center, or the velocity of the intact object as its center for the explosion component of the breakup. Specifically, for the 4 cases in Figure 2, there will be:

Case 1: the single object in the center of mass orbit

Case 2: an explosion cloud and a collision cloud, both centered on the center of mass velocity

Case 3: a single large object moving in its original orbit, and a debris cloud centered on the center of mass velocity for the directly involved collisional material

Case 4: an explosion debris cloud centered on the velocity of the large object and a debris cloud centered on the center of mass velocity of the directly involved collisional material.

The effect of momentum transfer in the collisional clouds is important because the center of mass velocity will be less than the circular orbit velocity, if neither of the initial velocities exceeds the circular orbit velocity. That means that in velocity space the center of the clouds will move toward the Z axis, and the fragments in the cloud will therefore have lower perigee altitudes than they would had momentum exchange not been accounted for.

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Looking at a debris cloud in velocity space, it is possible in a very straightforward way to determine the amount of material that re-enters almost immediately by calculating the volume of that cloud (in velocity space) having orbits inside a hyperboloid of low perigee altitude. An altitude of 200 km will be used in this paper. The volume will depend on the center of mass velocity for the cloud and on the characteristic velocity perturbations for the cloud particles.

Figure 4 presents the cross-section of a spherical debris cloud, showing velocity intervals of 200 m/sec up to 1 km/sec and centered on the circular orbit velocity. The shaded region represents the orbits that are reentering, where it must be noted that the full 3 dimensional space must be shown to measure the actual volume. The percentage of orbits reentering as a function of center of mass velocity is shown in Figure 5.

While the figures show how to represent the volume of the debris cloud in velocity space, the density distribution of fragments in this volume is the real quantity of interest. It is this density times the related volume that will characterize the number of objects reentering, or populating short- or long-life orbits. To calculate this density distribution, $N(v,d)$, the velocity distribution integrated over size must be established for the debris cloud.

This joint size and velocity distribution will be assumed to be separable and of form

$$N(v,d) = K_d \begin{cases} V/V_o & 0.1 V_o \leq V \leq V_o \\ \dfrac{1.3 - V/V_o}{0.3} & V_o \leq V \leq 1.3 V_o \end{cases} \quad (3)$$

as suggested by Kessler (/6/), where the K_d is evaluated from the size distribution. Integrating over all velocities leads to

$$N(d) = 0.645 \, K_d V_o \quad (4)$$

However, the expression for $N(d)$ is (/7/)

$$N(d) = bAm^{-(b+1)} \quad (5)$$

to give a value to K_d of

$$K_d = \frac{bAm^{-(b+1)}}{0.645 \, v_o} \quad (6)$$

To convert from mass to size, the relationship

$$m = \frac{\pi d^3 \rho}{6}$$

will be used, leaving K_d defined as

$$K_d = \frac{bA}{0.645} \left(\frac{\pi \rho}{6}\right)^{-(b+1)} \frac{d^{-3(b+1)}}{v_o(d)} \quad (7)$$

leading finally to a joint distribution function given by

$$N(v,d) = \frac{bA}{0.645} \left(\frac{\pi \rho}{6}\right)^{-(b+1)} \frac{d^{-3(b+1)}}{v_o(d)} \begin{cases} V/V_o & 0.1 V_o \leq V \leq V_o \\ \dfrac{1.3 - V/V_o}{0.3} & V_o \leq V \leq 1.3 V_o \end{cases} \quad (8)$$

The peak in the velocity distribution, denoted as v_0, is itself a function of size. In this paper, the size/velocity relationship derived by Su (/8/) is assumed. It is of form

$$\log_{10} V_o = \begin{cases} 0.875 - 0.676 \, (\log(d/d_m))^2 & d \geq d_m \\ 0.875 & d \leq d_m \end{cases} \quad (9)$$

where

$d_m = 9.9083 \times 10^{-8} m_p^{1/3} v_p^{2/3}$ (m)
m_p = projectile mass (kg)
v_p = impact velocity (km/s)

This function is plotted in Figure 6 for $m_p = 15kg$ and $v_p = 10km/s$. A line of constant velocity in this diagram will map onto a spherical surface in velocity space, when the center of the sphere is taken to be the velocity of the center of mass for the cloud. The relative contribution of different sizes of objects at a given breakup velocity v_1, i.e. the density distribution along a horizontal strip of Figure 6, can be seen in plots of $N(v_1,d)$.

For purposes of illustration throughout the rest of the paper, $d_m = 1.26 \times 10^{-6}$ meters and $b = 0.7496$ will be assumed. Figure 7 presents plots for $v_1 = 50m/s$, $300m/s$, and $1km/s$, where the curves have been normalized by dividing out the constant (size independent) part of K_d. This makes sense because only relative contributions are used in the following discussion. The $50m/s$ curve characterizes the largest objects in the breakup, the $300m/s$ velocity the centimeter fragments, and the $1km/s$ velocity the millimeter fragments.

To calculate the number of objects as a function of velocity only, $N(v_1)$, the joint distribution function must be integrated over size, thus

$$N(v_1) = \int_{D_{min}}^{D_{max}} N(v_1,x)\, dx \qquad (10)$$

where the upper and lower integration limits are functions of v_1. They are determined as follows:

(1) the lower limit is the diameter $D_{min}(v_1)$ having a v_0 satisfying $0.1 * v_0 = v_1$, and

(2) the upper limit is the diameter $D_{max}(v_1)$ having a v_0 satisfying $1.3 * v_0 = v_1$

Since the primary concern is for debris fragments that can seriously damage a spacecraft, D_{min} is taken to be no smaller than 1mm.

Performing the integral expressed in Equation 10 leads to the density distribution within the cloud. A plot of $N(v_1)$, using the same normalization as for Figure 7, is provided in Figure 8 for three cases - of 10cm and larger, 1cm and larger, and 1mm and larger debris clouds. Using this density distribution, the percentage of reentering objects is shown in Figure 9 for these three cloud components.

It only remains to relate the center of mass velocities, as derived from conservation of momentum, to the velocities shown on the abscissa of Figures 5, 8, and 9. This will be done for two cases in the following sections.

Case: Collision Induced by a Zero Velocity Projectile

For this case the collision is induced by a projectile near apogee in a ballistic orbit, so that the impact speed is the orbital speed of the target object. This is the type of ASAT test conducted against the Solwind satellite. It is the simplest case for calculating momentum transfer, as the results can be characterized by the single parameter of ratio of the projectile mass, m_p, to the column mass, m_c. The center of mass velocity as a function of these quantities is given by

$$V_{cm} = \frac{M_c}{M_c + M_p} V_\oplus = \frac{1}{1+X_0} V_\oplus \qquad (11)$$

where

$$x_0 = m_p / m_c$$

The ratio $m_c/(m_p + m_c)$ is plotted as a function of mass ratio, m_p/m_c, in Figure 10. The percentage of mass to re-enter as a function of mass ratio is provided in Figure 11.

Case: Collision Induced by a Projectile in Circular Orbit

The case of 2 objects in circular orbit colliding presents a more complex problem since the solutions depend on both mass ratio and encounter angle. The center of mass velocity, expressed in terms of these quantities, is

$$a_0 = \frac{1}{1+X_0} (1 + 2X_0 C_\Theta + X_0^2)^{1/2} \qquad (12)$$

where

$$C_x = Cos(x)$$
$$a_0 = v_{cm} / v_0$$

The greatest complication is that the collision speed, which enters the velocity distribution as seen in Equation 9, depends on this encounter angle through the simple relation

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$$V_p = \sqrt{2} \; V_\Theta \; (1 - C_\Theta)^{y_2} \qquad (13)$$

This has the effect of varying the distribution as shown in Figure 8 as a function of θ. For the volume of the cloud in velocity space that lies in the reentry region, as opposed to the number of reentering objects, this complication does not arise, and a plot such as Figure 12 can be used to characterize surfaces of constant ratio between the center of mass velocity and the orbital speed. The line of fixed a_0 corresponds to a single point on the horizontal axis of Figure 5; if the velocity distribution was not a function of θ, such a line would also correspond to a single point on the horizontal axis of Figure 9.

CONCLUSIONS

A two component collisional breakup process has been suggested to provide a mechanism for distinguishing between material directly involved in the collision process, and that material in the same structure only indirectly involved. Only the indirectly affected material forming an explosion-type of cloud has been observed in on-orbit tests, since this cloud contains the larger objects. Momentum transfer only involves the directly involved material, which is characterized by the column mass in the target. This model provides a method for identifying the mass involved in the collisional component of the resulting debris cloud.

Momentum transfer in the collisional component of a collisional breakup can lead to significant reduction in the amount of debris scattered into long-life orbits. Two cases were used to demonstrate the technique for determining center of mass velocities.

The major deficiency in the current work is that the effect of relative velocity, which is a parameter of the collisional debris velocity distribution, is not considered. Also, the suggested model decoupling the directly involved target mass from the residual mass can be better documented relative to hypervelocity impact tests and modeling than has been done in this paper. Both considerations are currently being addressed.

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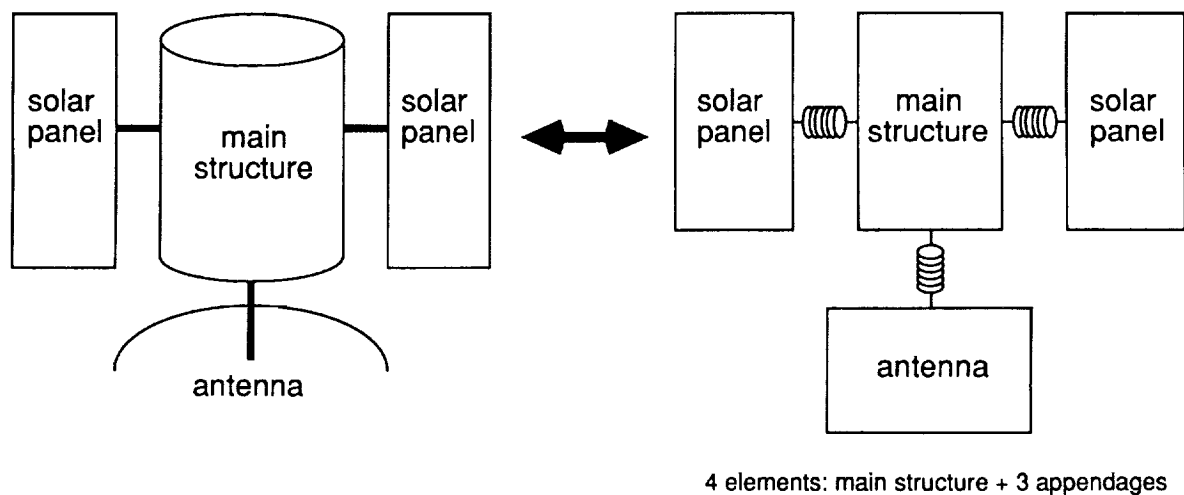
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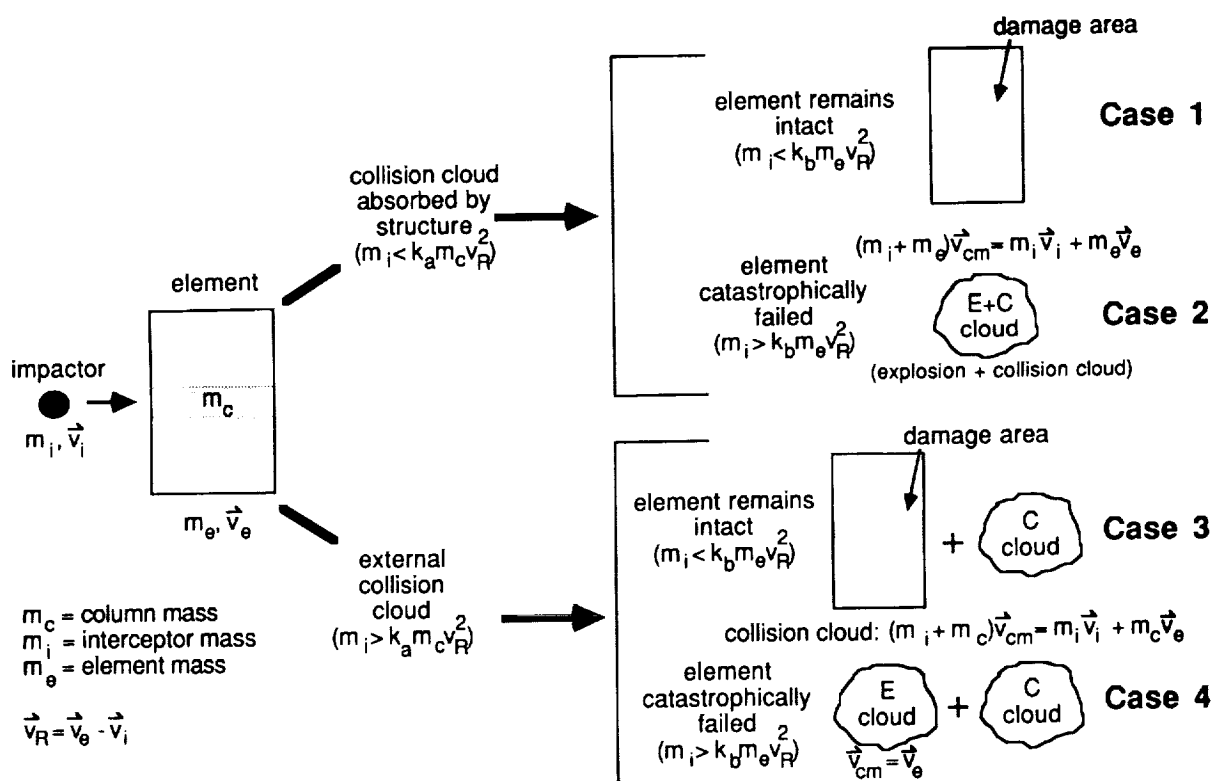FIGURE 1.   Element Diagram for Spacecraft

4 elements: main structure + 3 appendages

Case 1

element remains intact
$(m_i < k_b m_e v_R^2)$

damage area

Case 2

element catastrophically failed
$(m_i > k_b m_e v_R^2)$

$(m_i + m_e)\vec{v}_{cm} = m_i \vec{v}_i + m_e \vec{v}_e$

E+C cloud

(explosion + collision cloud)

collision cloud absorbed by structure
$(m_i < k_a m_c v_R^2)$

element

m_c

$m_e, \vec{v}_e$

impactor

$m_i, \vec{v}_i$

m_c = column mass
m_i = interceptor mass
m_e = element mass

$\vec{v}_R = \vec{v}_e - \vec{v}_i$

external collision cloud
$(m_i > k_a m_c v_R^2)$

Case 3

element remains intact
$(m_i < k_b m_e v_R^2)$

damage area

+ C cloud

collision cloud: $(m_i + m_c)\vec{v}_{cm} = m_i \vec{v}_i + m_c \vec{v}_e$

Case 4

element catastrophically failed
$(m_i > k_b m_e v_R^2)$

E cloud + C cloud

$\vec{v}_{cm} = \vec{v}_e$

FIGURE 2.   Diagram for Treating Collisional Interaction

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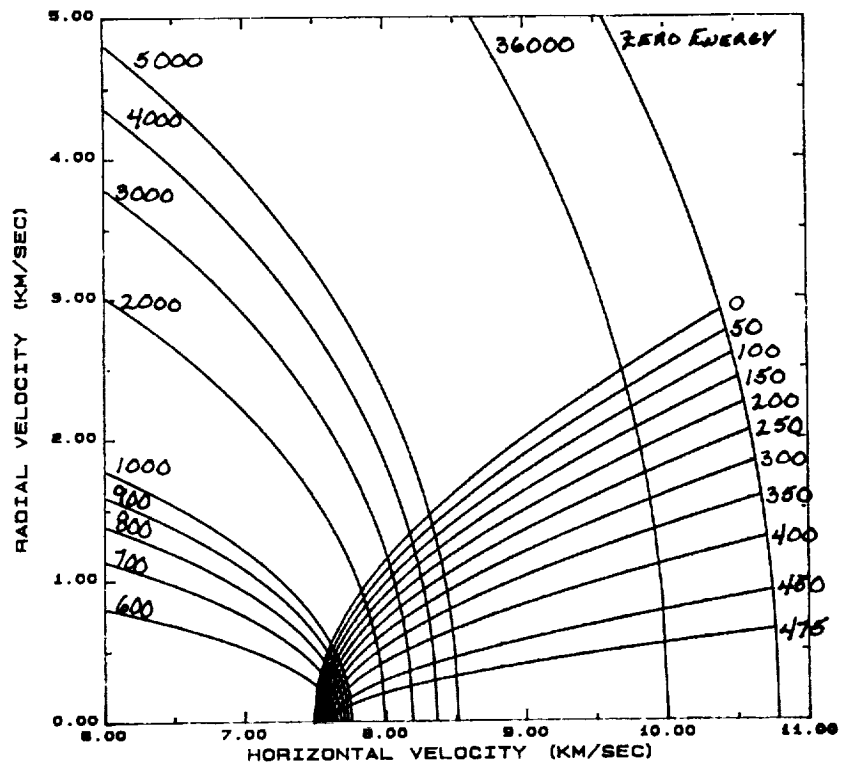
FIGURE 3.   Velocity Space Diagram for Altitude 500 Km.
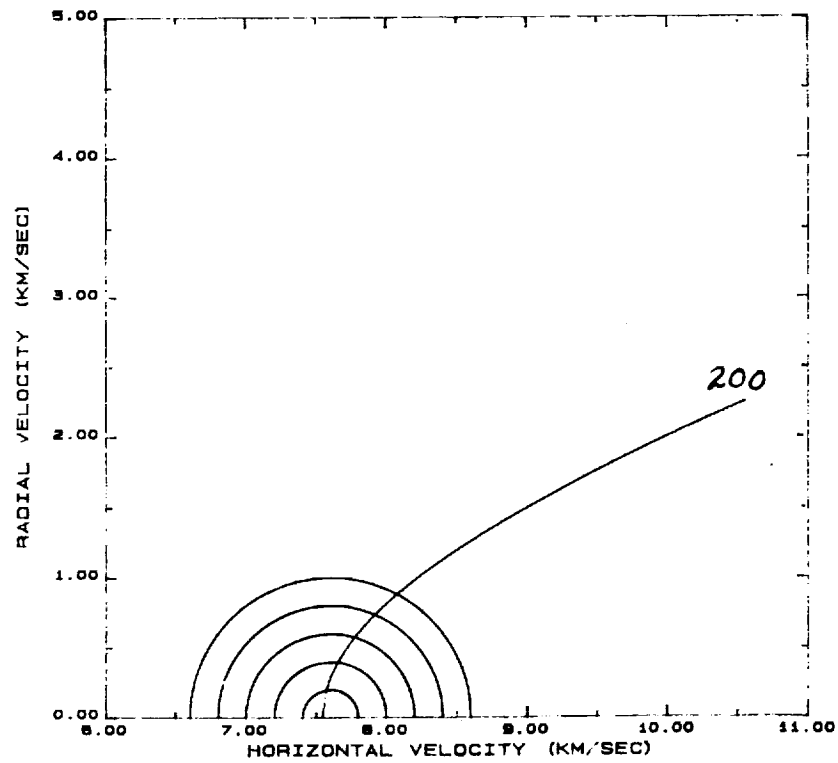
FIGURE 4.   Velocity Space Diagram for Altitude 500 Km - cloud included and shaded
region showing reentry orbits
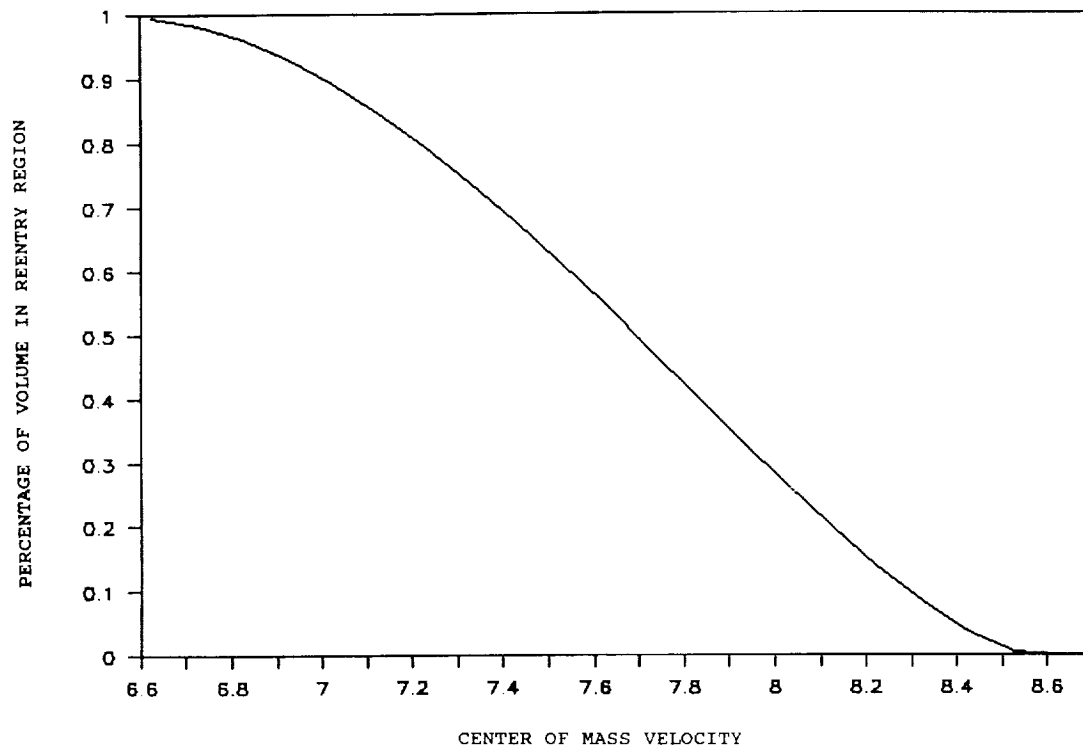
FIGURE 5.   Percentage of Velocity Space Cloud Volume in the Reentry Region vs.
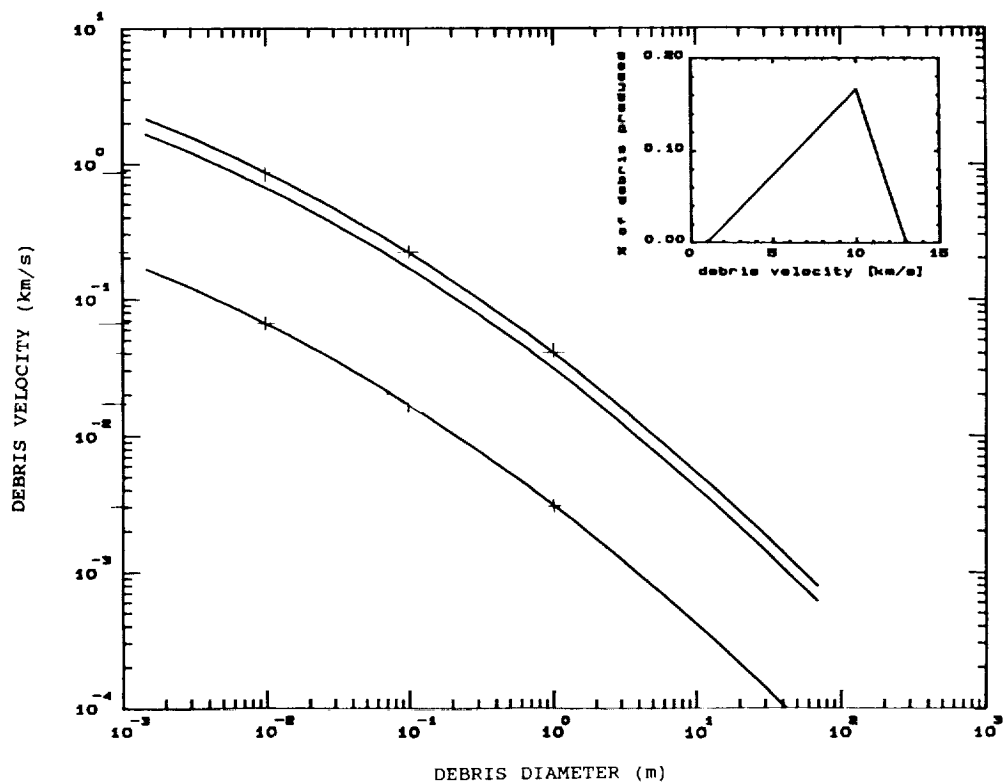Center of Mass Speed. Alt = 500 km.

FIGURE 6.   Breakup Velocity vs. Size Distribution for Projectile Mass of 15kg and
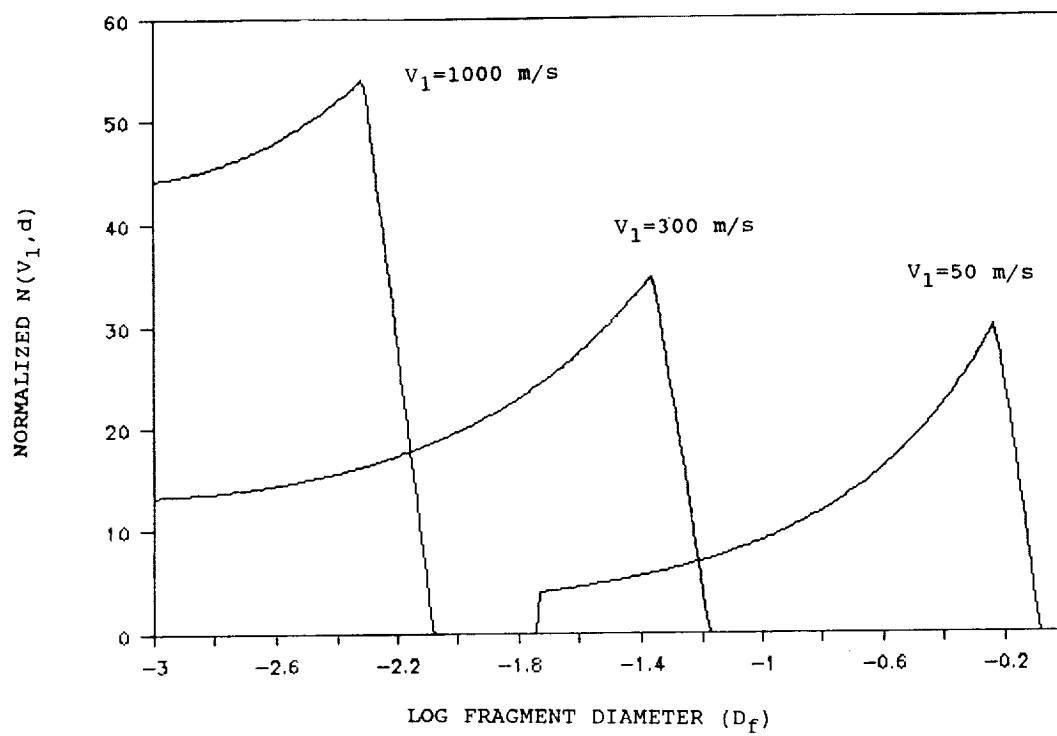Impact Velocity of 10km/s. (from /6/)

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FIGURE 7. Size Composition of Debris Cloud Components for Breakup Velocities of
50, 300, 1000 m/s Using the Reference Velocity and Size Distributions.
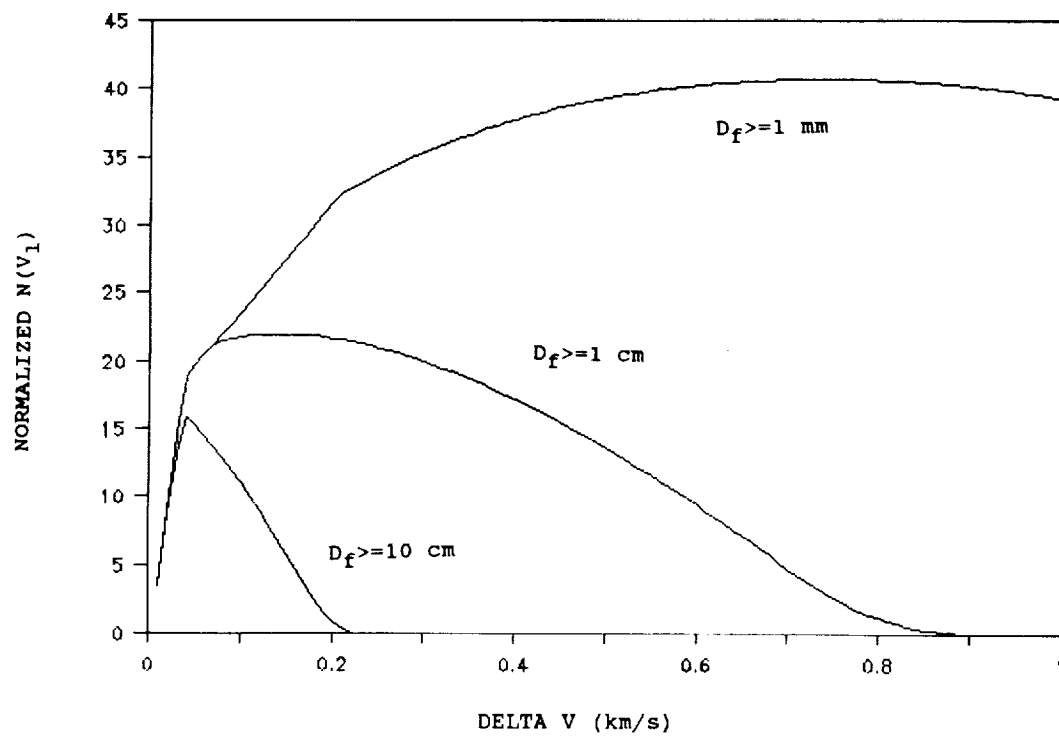
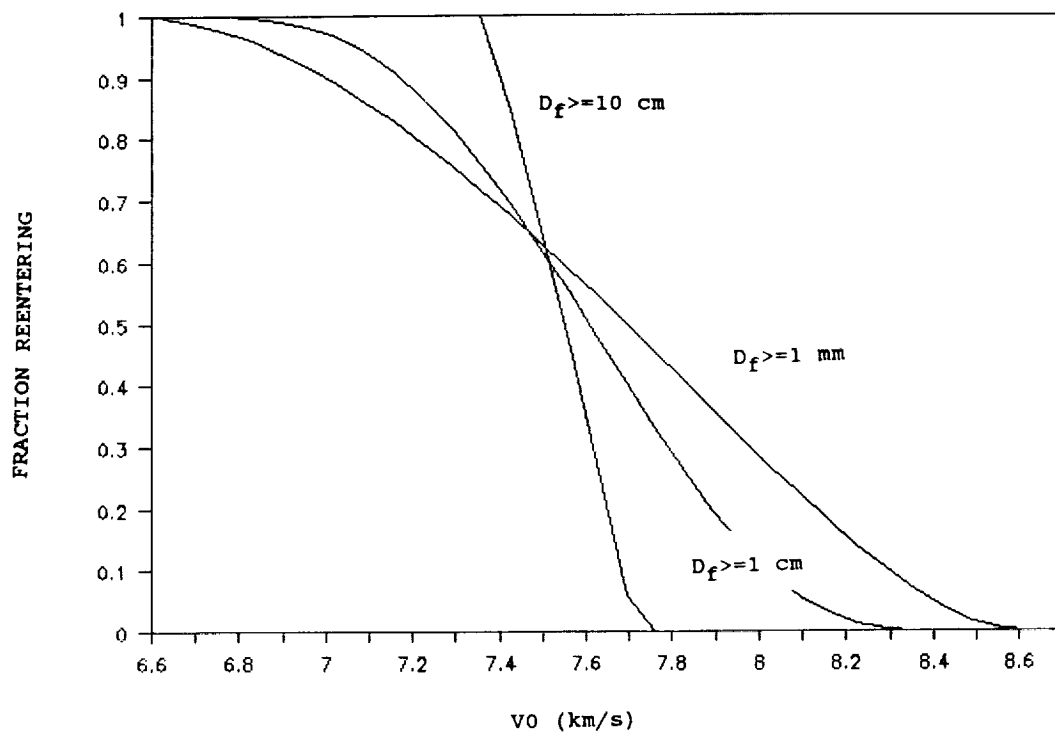FIGURE 8. Velocity Distribution for the Debris Cloud Fragments Larger Than 10cm,
1cm, and 1mm.

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FIGURE 9.   Percentage of Objects Reentering vs. Center of Mass Speed Using
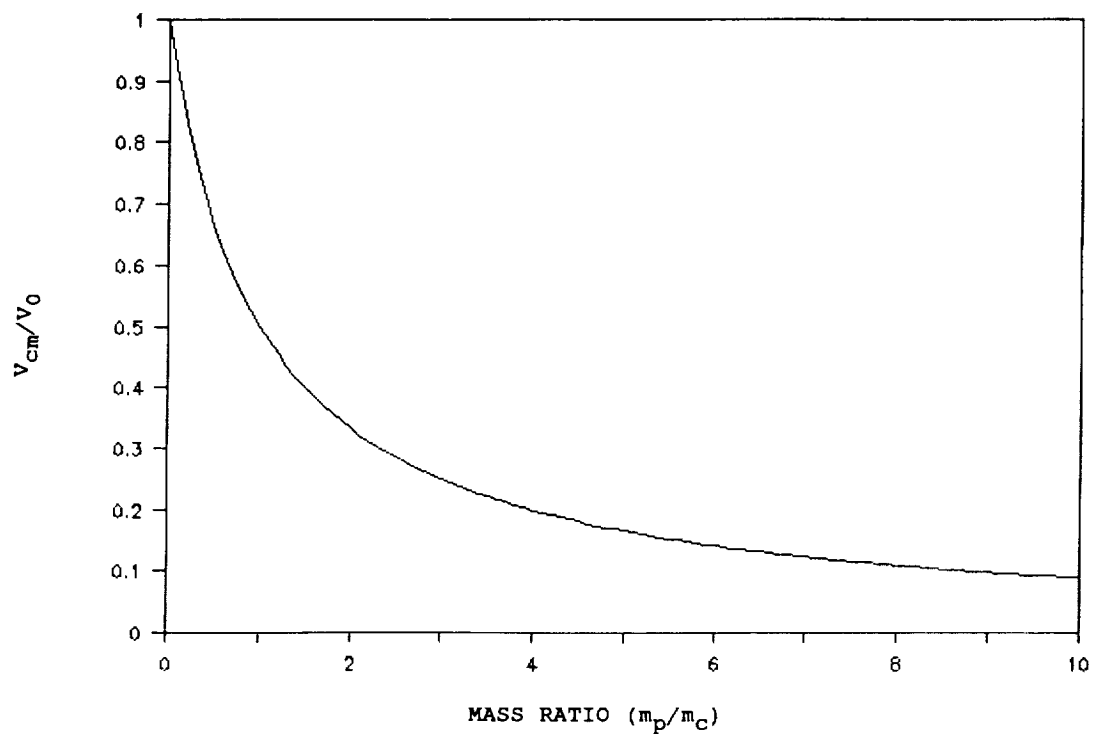Reference Velocity Distribution. Alt = 500 km.

FIGURE 10.   Hovering ASAT - Fraction of Circular Orbit Speed as a Function of
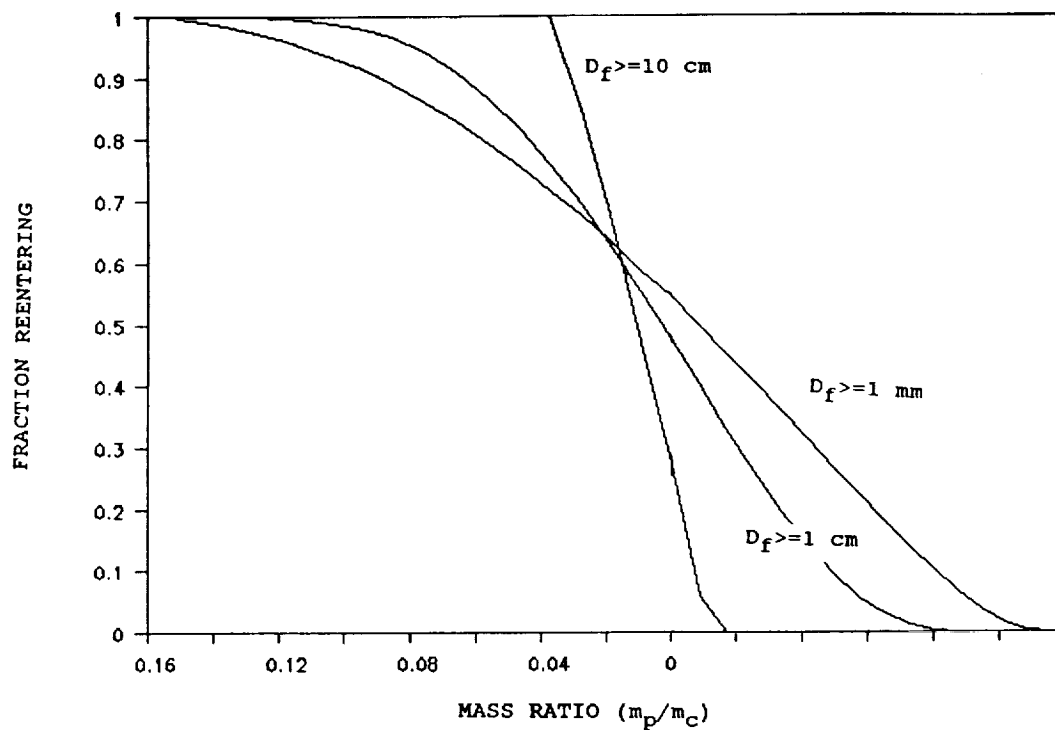Mass Ratio

FIGURE 11. Hovering ASAT - % of Objects to Re-enter as a Function of Mass Ratio.
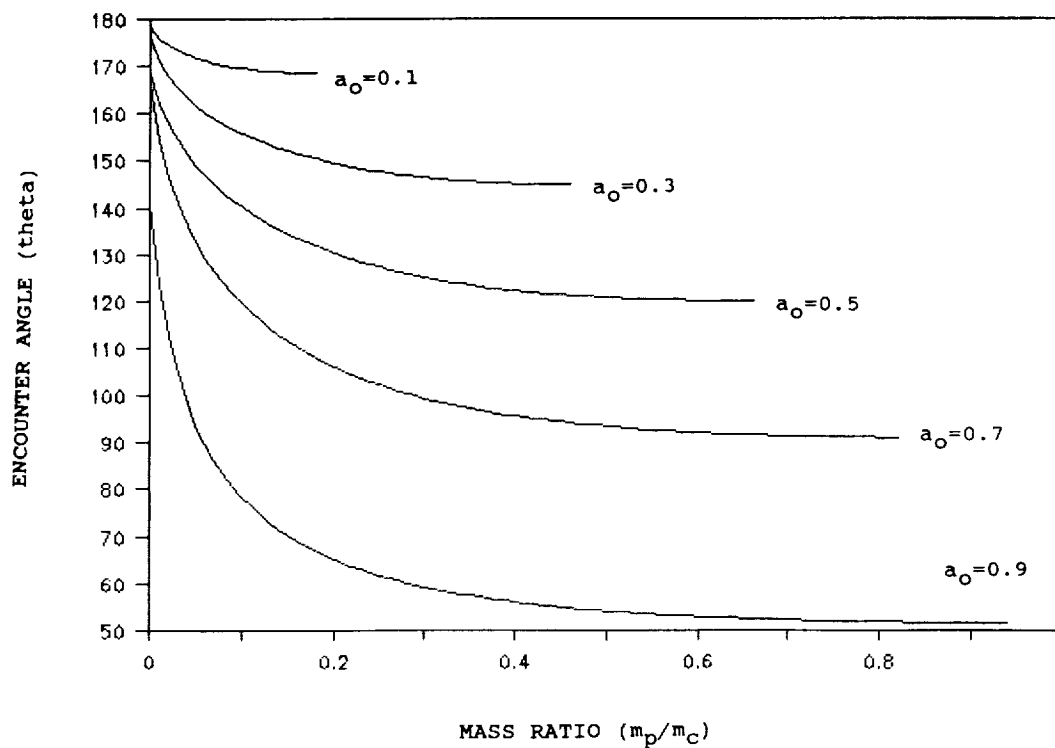Alt = 500 km.

FIGURE 12. Co-Orbiting ASAT - Contours of Constant Center of Mass Velocity as a
Function of Encounter Angle and Mass Ratio.

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